Since $\lim_{x \to a} \frac{f(x)-f(a)}{x-a} = f'(a)$ (i.e. the limits exists) and $\lim_{x \to a} x - a = 0$, we can break up (*) above by the Limit Laws

$$\lim_{x \to a}(f(x) - f(a)) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \cdot \lim_{x \to a}(x - a)$$
$$= f'(a) \cdot 0 = 0$$

Cautionary Remark Above theorem says differentiability $\Rightarrow$ continuity. But the converse is not true continuity $\not\Rightarrow$ differentiability. An example of this is $f(x) = |x|$ which is not differentiable at x=0 but it is continuous at zero.(More on this below)

Although there are many ways a function could fail to be differentiable at a point a, there are three typical types of non-differentiability.

(1)Discontinuities A function is not differentiable at a point where the graph of f is not continuous. This is a direct result of the above theorem and the remark following it.

(2)Corner A function is not differentiable at a point where the graph of f has a kink or corner (one might also call such a point on the graph a kink). Essentially, these places fail to be differentiable because the left and right hand limits

$$\lim_{x \to a^-} \frac{f(x) - f(a)}{x - a} \text{ and } \lim_{x \to a^+} \frac{f(x) - f(a)}{x - a}$$

do not match up. For instance, the absolute value function $f(x) = |x|$ fails to be differentiable at 0 because

$$\lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^-} \frac{|x| - |0|}{x} = \frac{|x|}{x} = \frac{-x}{x} = -1$$

and

$$\lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \frac{|x| - |0|}{x} = \frac{|x|}{x} = \frac{x}{x} = 1$$

A special case of a "corner" point non-differentiability is called a **"cusp"** where one of the one-sided limit above is ∞ and the other one-sided limit is $-\infty$. The picture below is an example of a cusp.