Example Find the derivative of the function $f(x) = x^2 - 8x + 9$ at the number a.

$$\begin{aligned}
\lim_{x\to a} \frac{f(x)-f(a)}{x-a} &= \lim_{x\to a} \frac{x^2-8x+9-(a^2-8a+9)}{x-a} \\
&= \lim_{x\to a} \frac{x^2-a^2-8x+8a}{x-a} \\
&= \lim_{x\to a} \frac{(x-a)(x+a)-8(x-a)}{x-a} \\
&= \lim_{x\to a} (x+a-8) \\
&= a+a-8 = 2a-8
\end{aligned}$$

Since the limit exists for any value of a, $f'(a) = 2a - 8$.

Example Use the second definition given in Remarks (2) to calculate $f'(a)$ for $f(x) = x^2 - 8x + 9$.

$$\begin{aligned}
f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} &= \lim_{h\to 0} \frac{(a+h)^2-8(a+h)+9-[a^2-8a+9]}{h} \\
&= \lim_{h\to 0} \frac{a^2+2ah+h^2-8a-8h+9-a^2+8a-9}{h} \\
&= \lim_{h\to 0} \frac{2ah-8h+h^2}{h} \\
&= \lim_{h\to 0} (2a-8+h) \\
&= 2a-8
\end{aligned}$$

Note the point "a" in the last two examples was arbitrary so we can declare that we found the derivative of f(x) at all points in its domain

$$f(x) = x^2 - 8x + 9 \text{ and } f'(x) = 2x - 8$$

Example Let $f(x) = x^2 - 8x + 9$. Find the equation of the tangent line to the curve $y = f(x)$ at the point $(3, -6)$.

Note that $f(3) = 3^2 - 8(3) + 9 = -6$. We also know that
$f'(3) =$ slope of the tangent line at $(3, f(3))$.
$f'(3) = 2(3) - 8 = -2$. So the equation of the tangent line through $(3, -6)$ with slope $m = -2$ is:

$$y - (-6) = (-2)(x - 3) \Rightarrow \boxed{y = -2x}$$