Sections 2.1/2.2 The Derivative

In developing the idea of the limit we talked about the derivative (recall sections 1.1/1.2) as

1) a rate of change or instantaneous velocity

2) the slope of the tangent line.

We now redefine it in terms of the limits without ϵ's and δ's.

Definition The derivative of a function f at a point "a", denoted by $f'(a)$ is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

if this limit exists and we say the function f is differentiable at $x = a$.

Remarks

1) This definition reinforces the geometric concept of the derivative

$$\begin{aligned}
\text{Slope of tangent line to f(x) at } (a, f(a)) &= f'(a) \\
&= \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \\
&= \text{ limit of slopes of secant lines through} \\
&\quad (a, f(a)) \text{ and } (x, f(x))
\end{aligned}$$

2) Other than the one we have seen in the definition there is a different but equivalent way to evaluate the slope of the tangent line or derivative of a function f(x) at a point x=a.
Let $h = x - a$ represent the distance between x and a. Since this gives $x = a + h$ you can substitute in the previous expression and get

$$\begin{aligned}
\text{Slope of tangent line to f(x) at } (a, f(a)) &= f'(a) \\
&= \lim_{h \to 0} \underbrace{\frac{f(a+h) - f(a)}{h}}_{\text{called difference quotient}}
\end{aligned}$$

Both of these expressions are equivalent ways to compute the derivative, and you should feel comfortable using at least one of them.