Rigorous Definition of one-sided limit We say that the right limit of f(x) as $x \to x_0$ is L if for any given $\epsilon > 0$ there is a $\delta > 0$ such that if $0 < x - x_0 < \delta$ then $|f(x) - L| < \epsilon$. And we write $\lim_{x \to x^+} f(x) = L$
Note that in the above definition $0 < x - x_0 < \delta$ replaced $|x - x_0| < \delta$ because as x approaches x_0 from right all x-values are bigger than x_0 hence $|x - x_0| = x - x_0$.

Similarly, we say that the left limit of f(x) as $x \to x_0$ is L if for any given $\epsilon > 0$ there is a $\delta > 0$ such that if $0 < x_0 - x < \delta$ then $|f(x) - L| < \epsilon$. And we write $\lim_{x \to x^-} f(x) = L$

Note that this time $0 < x_0 - x < \delta$ replaced $|x - x_0| < \delta$ because as x approaches x_0 from left all x-values are smaller than x_0 hence $|x - x_0| = x_0 - x$.