Use the above line to choose δ, choose δ so that the above result is less then ϵ. So I choose $\delta = \frac{\epsilon}{2}$. Hence;

$$|f(x) - 2| = |2x - 2| = 2|x - 1| < 2\delta = 2\frac{\epsilon}{2} = \epsilon$$

Example Show that $\lim_{x\to 2}(3x - 5) = 1$.

Given $\epsilon > 0$, we will choose $\delta > 0$ (how?):

$$\bullet |f(x) - 1| = |3x - 5 - 1| = |3x - 6| = 3|x - 2| < 3\delta$$

Last inequality follows by the fact that the δ determines the size of the neighborhood around 2, namely $|x - 2| < \delta$. Now if we choose $\delta = \frac{\epsilon}{3}$, $\bullet$ becomes

$$|f(x) - 1| = |3x - 5 - 1| = |3x - 6| = 3|x - 2| < 3\delta = 3\frac{\epsilon}{3} = \epsilon$$

Example Show that $\lim_{x\to 2} 2x^2 + x = 10$

Given $\epsilon > 0$, we will choose $\delta > 0$:

$$\begin{array}{rcc} |f(x) - 10| & = & |2x^2 + x - 10| \\ & \leq & |2x^2 + x + 10| \\ & = & |(2x + 5)(x - 2)| \\ & = & |2x + 5||x - 2| \\ & < & |2x + 5|\delta \\ & = & \delta(|2(x - 2) + 9|) \\ & \leq & \delta(2|(x - 2)| + |9|) \\ & < & \delta(2\delta + 9) \\ & < & \delta(2 \cdot 1 + 9) \text{ (assuming } \delta < 1) \\ & = & 11\delta \end{array}$$

So now choose $\delta = Min(1, \frac{\epsilon}{11})$. So the last line above will be then $< \epsilon$.

Example Show that $\lim_{x\to 0} \sin(x) = 0$

Given ϵ, we will choose $\delta > 0$ such that when $|x| < \delta$ $|f(x) - L| = |\sin(x) - 0| = |\sin(x)| \leq |x|$ for$0 \leq x \leq 1$ So we found our desired quantity less than the "length we control", the length of $|x|$. Now simply choose $\delta = \epsilon$ and hence proved that $|f(x) - L| < \epsilon$.