Sections 1.6 Some Limit Proofs

Let's go back and using the definition of the limit prove couple of limits.

Theorem For any constant c and any real number a, $\boxed{\lim_{x\to a} c = c}$

Proof Let $\epsilon > 0$ be given, now we need to choose a $\delta > 0$ such that for any x, that satisfies $|x - a| < \delta$, the condition $|f(x) - c| < \epsilon$ is satisfied. Well this one is an easy one to prove because $f(x) = c$ for any x. So $|f(x) - c| = |c - c| = 0$. So there is no game here at all, because for any given $\epsilon > 0$, it doesn't matter what δ I choose, for any x, in any $(a-\delta, a+\delta)$ interval corresponding $|f(x) - c| = |c - c| = 0 < \epsilon$. So without a sweat in this case $\lim_{x\to a} c = c$.

Example $\lim_{x\to 3} 44 = 44$

Theorem For any real number a, $\boxed{\lim_{x\to a} x = a}$.

Proof Let $\epsilon > 0$ be given. My aim as before is, to find a $\delta > 0$ such that when $|x - a| < \delta$, $|f(x) - a| < \epsilon$. Start with your goal $|f(x) - a|$.

$|f(x) - a| = |x - a|$ because $f(x) = x$. So another easy one. If I choose my $\delta = \epsilon$ I am done:

$$|x - a| < \delta \Rightarrow |f(x) - a| = |x - a| < \delta = \epsilon$$

Example $\lim_{x\to 5} x = 5$

Example Show that $\lim_{x\to 1} 2x = 2$.

Given any $\epsilon > 0$, we need to find our $\delta > 0$. We will decide on that choice depending what we are trying to show.So start with your destination:

$|f(x)-2| = |2x-2| = 2|x-1|$ (by using the abs. value property $|ab| = |a||b|$

So the inequality I will try to show less than the given ϵ, is depended on my choice of δ directly, as with δ I'm controlling the length of $|x - 1| < \delta$.

$$|f(x) - 2| = |2x - 2| = 2|x - 1| < 2\delta$$