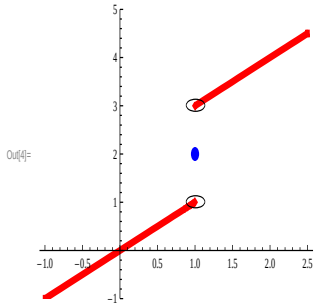
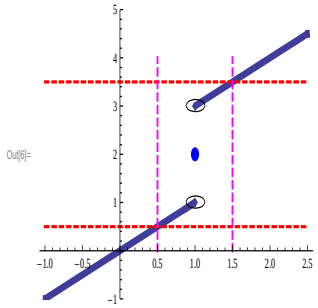
Let's see an example where you get to win:

Example Find the $lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} x & x < 1 \\ 2 & x = 1 \\ x+2 & x > 1 \end{cases}$

The graph of $f(x)$ is given also below.



Assume I claim $\lim_{x\to 1} f(x) = 2$.Now if you start with an ϵ such as $\epsilon = 1.5$. I will easily choose my $\delta = 1/2$ and according to the graph below I will win:



BUT if you choose your $\epsilon = 0.5$ then there is no $\delta > 0$ I can choose. So $\lim_{x\to 1} f(x)$ does not exits. Observe below couple of my failed attempts: