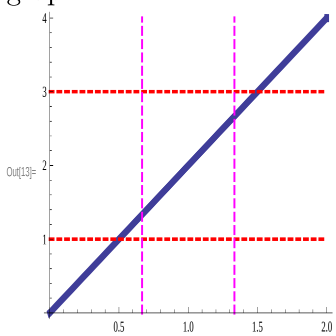
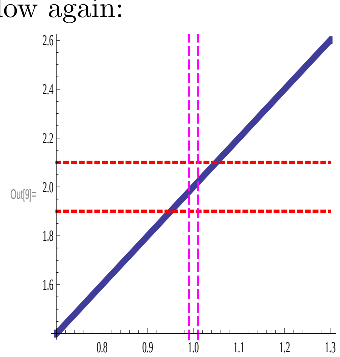
Example Claim: $\lim_{x\to 1} 2x = 2$. So the game is on. Let say first you throw at me $\epsilon = 1$. This means you have opened an $\epsilon = 1$ unit wide rectangle around L=2 on the y-axis and challenged me to define my nearness criterion. I choose $\delta = 1/3$ (don't pay attention how I choose mine for the time being just watch how the game is played.). Now let's see who is the winner of this round. If I have chosen mine good, for all x values that are 1/3 units away from 1, ($|x-1| < 1/3$ means just that) all the corresponding $f(x) = 2x$ values distance to 2 should not exceed your $\epsilon = 1$. Check out the following graph:



It looks like I win this round. The part of $f(x) = 2x$ that falls within my purple window is shorter than the portion in your red window.

We are not done. You have not exhausted all your possible ϵ choices yet. Maybe you were too nice to me with your first choice choose a smaller ϵ, such as $\epsilon = 1/10$. I'll immediately choose my $\delta = 1/10^2$ check the graph below again:



According to the graph I win again. The part within my purple window is shorter than the part in your red window. If we continue the game like this soon you will realize that no matter what ϵ you give me, I can respond with a δ that works. Once, it is mutually recognized that, for every possible ϵ, I can counter with a δ we declare that $\lim_{x\to 1} 2x = 2$.