Sections 1.6 Rigorous Definition of Limit

Some recall: We have talked about "derivative" in Section 1.1/1.2. We have seen two different interpretations of this concept:

1. physically as instantaneous speed

2. geometrically as the slope of the tangent line.

	Physics	Geometry
f(a)	A function that gives the displacement of a particle from a fixed origin	The graph consisting of the point $(a, f(a))$
$\frac{f(x)-f(a)}{x-a}$	The average velocity of the particle on the time interval [a,x]	The slope of the secant line between the points $(a, f(a))$ and $(x, f(x))$
$f'(a) = \frac{df}{dx}\|_a$	The instantaneous speed of the particle at time a	The slope of the tangent line to the graph at the point (a,f(a))

We have also said that $f'(a) \approx \frac{f(x)-f(a)}{x-a}$ for x close to a. (Instantaneous speed is the "limit" of the average speeds of the particle in the time interval [a,x]). To be more mathematically correct we have been thinking

$$\frac{df}{dx}(a) = \underbrace{\frac{f(x) - f(a)}{x - a}}_{\text{average rate of change}} + \underbrace{E(x, a)}_{Error}$$

and assuming the error term is going to zero. As we have noted in the GPS problem the difference between these two quantities – the instantaneous and the average one – is crucial. (The difference between skipping the tip of the mountain or just crushing into it on an airplane !!). Hence we need to be more precise ....

Precision will come with a better understanding of the definition of the limit.