Example Find $\lim_{x\to\infty} \frac{3x-7}{\sqrt{5x^2+4}}$ and $\lim_{x\to-\infty} \frac{3x-7}{\sqrt{5x^2+4}}$.

$$\lim_{x\to\infty} \frac{3x-7}{\sqrt{5x^2+4}} = \lim_{x\to\infty} \frac{3x-7}{\sqrt{x^2(5+\frac{4}{x^2})}} = \lim_{x\to\infty} \frac{3x-7}{|x|\sqrt{5+\frac{4}{x^2}}}$$

At this point realize that when you say x is going to infinity you are dealing with very large positive numbers. So for these x values $|x| = x$. Hence add this extra info into your limit discussion above to get

$$\lim_{x\to\infty} \frac{3x-7}{\sqrt{5x^2+4}} = \lim_{x\to\infty} \frac{3x-7}{|x|\sqrt{5+\frac{4}{x^2}}} = \lim_{x\to\infty} \frac{3x-7}{x\sqrt{5+\frac{4}{x^2}}} = \lim_{x\to\infty} \frac{3-\frac{7}{x}}{\sqrt{5+\frac{4}{x^2}}} = \frac{3}{\sqrt{5}}$$

For the other limit (negative infinity) we will have the same beginning steps up to the point where we decide the sign of the $|x|$. This time because x goes to negative infinity means you are dealing with very "large" negative numbers $|x| = -x$. So you will proceed in your calculations as below:

$$\lim_{x\to-\infty} \frac{3x-7}{\sqrt{5x^2+4}} = \lim_{x\to-\infty} \frac{3x-7}{|x|\sqrt{5+\frac{4}{x^2}}} = \lim_{x\to-\infty} \frac{3x-7}{-x\sqrt{5+\frac{4}{x^2}}} = -\frac{3}{\sqrt{5}}$$

So this function has two different horizontal asymptotes one at $y = \frac{3}{\sqrt{5}}$ and the other at $y = -\frac{3}{\sqrt{5}}$

Example Find $\lim_{x\to\infty} \sqrt{x+\sqrt{x}} - \sqrt{x}$

This would be a classic example of using the conjugate of an expression. The conjugate of $\sqrt{x+\sqrt{x}} - \sqrt{x}$ is $\sqrt{x+\sqrt{x}} + \sqrt{x}$. So;

$$\lim_{x\to\infty} (\sqrt{x+\sqrt{x}} - \sqrt{x}) \cdot \frac{\sqrt{x+\sqrt{x}}+\sqrt{x}}{\sqrt{x+\sqrt{x}}+\sqrt{x}} = \lim_{x\to\infty} \frac{x+\sqrt{x}-x}{\sqrt{x+\sqrt{x}}+\sqrt{x}} = \lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x}+\sqrt{x}}}$$

Now you need the old trick the highest power of x in the denominator is $\sqrt{x}$ so divide both numerator and denominator by this term:

$$\lim_{x\to\infty} (\sqrt{x+\sqrt{x}} - \sqrt{x}) = \lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x}+\sqrt{x}}} \cdot (\frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}}) = \lim_{x\to\infty} \frac{1}{\sqrt{\frac{1}{\sqrt{x}}+1+1}} = \frac{1}{2}$$