

**Example** Find  $\lim_{x \rightarrow \infty} \frac{x+1}{x^2}$ .

Once again divide both the denominator and the numerator by the highest power of  $x$  that appears in the denominator. This time it is  $x^2$ ;

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x+1}{x^2} &= \lim_{x \rightarrow \infty} \frac{x+1}{x^2} \cdot \left(\frac{1/x^2}{1/x^2}\right) \\ &= \lim_{x \rightarrow \infty} \frac{\frac{1}{x} + \frac{1}{x^2}}{1} \\ &= \frac{\lim_{x \rightarrow \infty} \frac{1}{x} + \lim_{x \rightarrow \infty} \frac{1}{x^2}}{\lim_{x \rightarrow \infty} 1} \\ &= \frac{0}{1} = 0 \end{aligned}$$

By comparing with the three examples above the following result for infinite limits of rational numbers should be clear:

**Theorem** Let  $R(x) = \frac{a_m x^m + a_{n-1} x^{m-1} + \dots + a_1 x + a_0}{b_n x^n + b_{n-1} x^{n-1} + \dots + b_1 x + b_0}$  be a rational function then

$$\lim_{x \rightarrow \infty} R(x) = \begin{cases} \infty & \text{if } m > n \\ \frac{a_m}{b_n} & \text{if } m = n \\ 0 & \text{if } m < n \end{cases}$$

So the answers of the following example should be clear:

**Example**  $\lim_{x \rightarrow \infty} \frac{(2x-3)^{20}(3x+2)^{30}}{(2x+1)^{50}} = \lim_{x \rightarrow \infty} \frac{2^{20}3^{30}}{2^{50}} = \left(\frac{3}{2}\right)^{30}$

How about some irrational function examples :

**Example** Find  $\lim_{x \rightarrow \infty} \frac{3x-2|x|}{x+2}$  and  $\lim_{x \rightarrow -\infty} \frac{3x-2|x|}{x+2}$

Note that  $f(x) = \frac{3x-2|x|}{x+2} = \begin{cases} \frac{3x-2x}{x+2} = \frac{x}{x+2} & \text{if } x > 0 \\ \frac{3x+2x}{x+2} = \frac{5x}{x+2} & \text{if } x < 0 \end{cases}$  So when discussing the infinite limits you need to consider two different functions:

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{3x-2|x|}{x+2} &= \lim_{x \rightarrow \infty} \frac{x}{x+2} = 1 \text{ because } m=n=1 \text{ and } a_1 = b_1 = 1 \\ \lim_{x \rightarrow -\infty} \frac{3x-2|x|}{x+2} &= \lim_{x \rightarrow -\infty} \frac{5x}{x+2} = \lim_{x \rightarrow -\infty} \frac{5}{1+\frac{2}{x}} = \frac{5}{1} = 5. \end{aligned}$$