Example Find $\lim_{x\to\infty} \frac{x+1}{x}$
A standard technique is to divide both the denominator and the numerator by the highest power of x that appears in the denominator. In this case, this means x:

$$\begin{aligned}
\lim_{x\to\infty} \frac{x+1}{x} &= \lim_{x\to\infty} \frac{x+1}{x} \cdot (\frac{1/x}{1/x}) \\
&= \lim_{x\to\infty} \frac{1+\frac{1}{x}}{1} \\
&= \frac{\lim\limits_{x\to\infty} 1 + \lim\limits_{x\to\infty} \frac{1}{x}}{\lim\limits_{x\to\infty} 1} \\
&= \frac{1+0}{1} = 1
\end{aligned}$$

You could also reach the same conclusion by re-writing $f(x) = \frac{x+1}{x} = 1 + \frac{1}{x}$ and recognizing that ax x goes to infinity the term $\frac{1}{x}$ is really small compared to 1, so the long run behavior of this function is more depended on the 1 part of the function rather than the $\frac{1}{x}$. Also note that by our definition of horizontal asymptotes $x = 1$ is a horizontal asymptote for f.

Example Find $\lim_{x\to\infty} \frac{x^2+1}{x}$.
Once again divide both the denominator and the numerator by the highest power of x that appears in the denominator. Again it is x;

$$\begin{aligned}
\lim_{x\to\infty} \frac{x^2+1}{x} &= \lim_{x\to\infty} \frac{x^2+1}{x} \cdot (\frac{1/x}{1/x}) \\
&= \lim_{x\to\infty} \frac{x+\frac{1}{x}}{1} \\
&= \frac{\lim\limits_{x\to\infty} x + \lim\limits_{x\to\infty} \frac{1}{x}}{\lim\limits_{x\to\infty} 1}
\end{aligned}$$

Right there we stop. Even though in an extended real line course you can write that limit directly as $\frac{\infty+0}{1} = \infty$, we cannot do that in this class and we don't need to. Think of this way at the end you are considering $\lim\limits_{x\to\infty} x + \frac{1}{x}$. Just as we noted at the end of the previous example as x gets larger and larger indefinitely, so does the x term in this limit: Since $\frac{1}{x}$ is small when x is large, it will not affect the value of x by much so $x + \frac{1}{x}$ will become very large and hence the limit is ∞.