

**How do we calculate limits at infinity?** Well, three things.

- 1) The Limit Laws and the Squeeze Theorem remain valid for limits at infinity. The usual restriction applies: For the limit of a quotient to be expressible as the quotient of the limits, the denominator needs to be nonzero. We need to add in another type of condition. I mentioned before that  $\infty$  and  $-\infty$  are not real numbers. They do not denote anything on the real number line. Rather they are symbols to help us express the idea that a variable, say  $x$ ; is taking on indefinitely large (or large negative) values. Thus we should not be using it, at least at this level, for arithmetical calculations. There is nothing called  $\infty/\infty$  for example. So, we will not say the limit of a quotient is the quotient of the limits in case the limit of the top or that of the bottom happens to be  $\pm\infty$ . This, however, does not mean that we can't say anything about limits of the structures in which some components involve  $\pm\infty$ . (More on this later)
- 2) Besides the obvious limit  $\lim_{x \rightarrow \infty} k = \lim_{x \rightarrow -\infty} k = k$  for any constant  $k$ , we have another basic limit fact (i.e., basic building block):

$$\lim_{x \rightarrow \infty} \frac{1}{x^p} = 0 \text{ and } \lim_{x \rightarrow -\infty} \frac{1}{x^p} = 0$$

for all positive powers  $p$  and  $\frac{1}{x^p}$  allows us to talk about the limit. What does this last sentence "the domain of  $\frac{1}{x^p}$  allows us to talk about the limit" mean? Well, let  $p = 1/2$  and consider  $\lim_{x \rightarrow -\infty} \frac{1}{x^{1/2}}$ . Even discussing such a limit is absurd because for negative values of  $x$ ,  $\sqrt{x}$  is not defined so we cannot talk about a limit, because  $\frac{1}{x^{1/2}}$  is not a function. So the above limit facts are true only if it makes mathematical sense to talk about it.

- 3) A third basic limit you will need is the polynomials: For any  $k > 0$

$$\lim_{x \rightarrow \infty} x^k = \infty \text{ and } \lim_{x \rightarrow -\infty} x^k = \begin{cases} +\infty & \text{if } k \text{ is even} \\ -\infty & \text{if } k \text{ is odd} \end{cases}$$