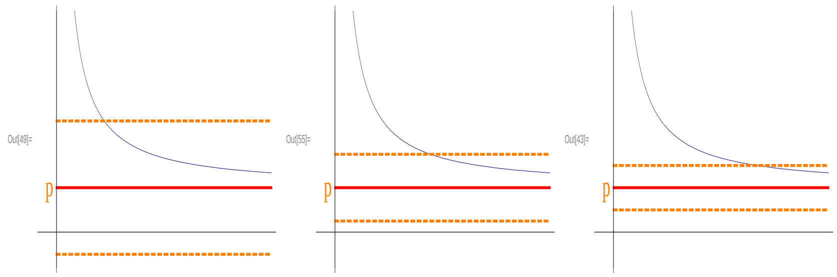
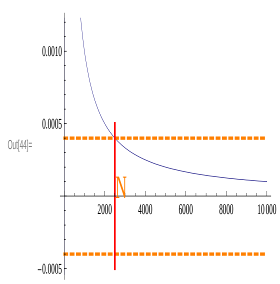
Intuitively $\lim_{x\to\infty} f(x) = p$ means that the graph lies in a horizontal strip around y=p for all sufficiently large positive x values. And as the width of this strip decreases the graph of $y = f(x)$ approaches the horizontal line $y = p$ really close. You can observe this in the graphs below.



Example Let $f(x) = \frac{1}{x}$ and consider $\lim_{x\to\infty} \frac{1}{x}$.

As $x \to \infty$ f(x) goes to zero or $\lim_{x\to\infty} \frac{1}{x} = 0$ because as the denominator of the fraction gets larger the fraction itself becomes really small. Note though it is never equal to zero, but by choosing x large enough f(x) can be made as close to zero as we would like. More formally let $\epsilon > 0$. There is a number N such that $|\frac{1}{x} - 0| < \epsilon$ for $x > N$; that is once you pass $x = N$ marker, your function values will be always within the strip around zero just like in the graph below.



This brings us to the idea of Horizontal Asymptotes.

Definition The line $y = p$ (or $y = q$) is called a horizontal asymptote of the curve $y = f(x)$ if either $\lim_{x\to\infty} f(x) = p$ (or $\lim_{x\to-\infty} f(x) = q$).
So in the above example $y = 0$ is a horizontal asymptote for $f(x) = \frac{1}{x}$.