$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}(x+1)} = \lim_{x\to 1} \frac{x+2}{x+1} = \frac{3}{2}$$

So $x = 1$ is not an vertical asymptote for $f(x) = \frac{x^2+x-2}{x^2-1}$.

Example Find $\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{-1}{(x-1)^4}$.

Here at $x = 1$ $p(1) = -1 \neq 0$ and $q(1) = (1-1)^4 = 0$ so by the rule above $x = 1$ is a vertical asymptote for this function. To be exact $\lim_{x\to 1} \frac{-1}{(x-1)^4} = -\infty$. Check this result creating the graph of f just like we did above for $\frac{-1}{|x-1|}$.

Limits at Infinity

In applications one is often interested in the asymptotic behavior of a function or in what the function does in the long run. That is $\lim\limits_{x\to\infty} f(x)$ or $\lim\limits_{x\to-\infty} f(x)$.

Definition

i) We say that as x approaches infinity f(x) approaches p, written $\lim_{x\to\infty} f(x) = p$, if for every ϵ there is an $M > 0$ such that if $x > M$ then $|f(x)-p| < \epsilon$

ii) We say that as x approaches negative infinity f(x) approaches q, written $\lim_{x\to-\infty} f(x) = q$, if for every ϵ there is a negative number -M such that if $x < -M$ then $|f(x) - q| < \epsilon$