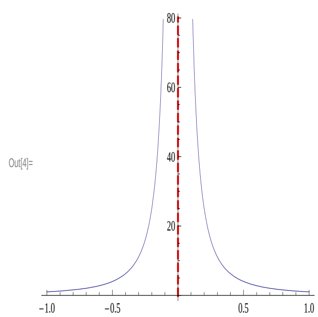
Informally $\lim_{x\to c} f(x) = \infty$ says that the function can be made arbitrarily large by making x sufficiently close to c. (or in the $-\infty$ case can be made arbitrarily small by making x sufficiently close to c.)

Definition The line $x = c$ is called a vertical asymptote of the curve $y = f(x)$ if at least one of the following is true.

$$\lim_{x\to c^-} f(x) = \infty \text{ or } \lim_{x\to c^-} f(x) = -\infty \text{ or } \lim_{x\to c^+} f(x) = \infty \text{ or } \lim_{x\to c^+} f(x) = -\infty$$

Example Find $\lim_{x\to 0} \frac{1}{x^2}$.

Check out the graph and the table corresponding to function $f(x) = \frac{1}{x^2}$ when x is close to zero



x	x^2	$\frac{1}{x^2}$		x	x^2	$\frac{1}{x^2}$
$0.1=10^{-1}$	10^{-2}	100		-10^{-1}	10^{-2}	100
10^{-2}	10^{-4}	10,000		-10^{-2}	10^{-4}	10,000
10^{-3}	10^{-6}	1,000,000		-10^{-3}	10^{-6}	1,000,000
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$
10^{-12}	10^{-24}	10^{24}		-10^{-12}	10^{-22}	10^{24}
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$

So based on both, the graph and the table the function $\frac{1}{x^2}$ grows without a bound no matter from which direction you approach to zero (left or right), hence $\lim_{x\to 0^+} \frac{1}{x^2} = \infty$ and $\lim_{x\to 0^-} \frac{1}{x^2} = \infty$. So $\lim_{x\to 0} \frac{1}{x^2} = \infty$. Hence $x = 0$ is vertical asymptote for $f(x) = \frac{1}{x^2}$ by definition above.

Example Find $\lim_{x\to 1} \frac{-1}{|x-1|}$

Let's see how to get the graph of $\frac{-1}{|x-1|}$ starting with the graph of $\frac{1}{x}$ first. After all so far graphs have been our most helpful tool to find the limit.