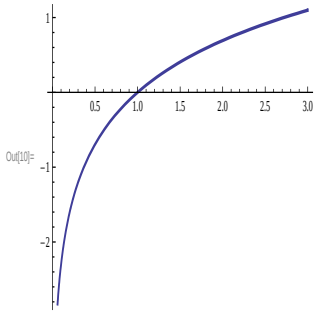
Sections 1.5 Limits involving Infinity

Limits to Infinity

Example: Consider $f(x) = \ln(x)$



As x approaches zero from the right hand side ($x \to 0^+$), $\ln(x)$ grows unboundedly in the negative direction. So if we are considering $\lim_{x\to 0^+} \ln(x)$ what is the answer? Since there is no finite number L such that this limit is equal to; your book and some other sources claims this limit to be non-existing (or limit does not exist). But this definition is not as informative about the behavior of the function.So if and whenever possible I would like you to state the exact destinations of your limits such as in this case $\lim_{x\to 0^+} \ln(x) = -\infty$. Having said that I do not disagree with the claim that this limit does not exist because $+\infty$ and $-\infty$ are not part of the real line but the curve sketching part we will cover later in the course will require us to look at the "non-existing" issue closer so why not start getting used to it now.

Definition

i) We say that $f(x)$ approaches infinity as x approaches to c and we write $\lim_{x\to c} f(x) = \infty$ if for every positive number $B > 0$ there exists a corresponding $\delta > 0$ such that for all x satisfying $0 < |x-c| < \delta \Rightarrow f(x) > B$

ii) We say that $f(x)$ approaches negative infinity as x approaches to c and we write $\lim_{x\to c} f(x) = -\infty$ if for every negative number $-B$ there exists a corresponding $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta \Rightarrow f(x) < -B$