Specifically, we notice that finding a number x with $x = \cos(x)$ is the same as finding a solution to the equation $f(x) = 0$, where $f(x) = \cos(x) - x$. Why is this true? A solution to $f(x) = 0$ is a number so that $\cos(x) - x = 0$, or equivalently, so that $\cos(x) = x$. This is exactly the condition we want!

Great, so lets try to show that for the function $f(x) = \cos(x) - x$ there exists a solution to $f(x) = 0$. Since we know we have to use the intermediate value theorem, we start by observing that f(x) is the difference of two continuous functions, and is therefore itself continuous. Furthermore we can see that

$$f(0) = \cos(0) - 0 = 1 - 0 = 1 > 0$$
$$f(\frac{\pi}{2}) = \cos(\frac{\pi}{2}) - \frac{\pi}{2} = 0 - \frac{\pi}{2} = -\frac{\pi}{2} < 0$$

Therefore, since f(x) is a continuous function on $[0, \frac{\pi}{2}]$ with $f(0) > 0$ and $f(\frac{\pi}{2}) < 0$, we may apply the IVT and conclude that there exists a number c in the interval $(0, \frac{\pi}{2})$ with $f(c) = 0$. This magical value of c therefore satisfies $\cos(c) - c = 0$, and so $\cos(c) = c$. Thats just what we want.