IVT has many useful applications. We will see two of them.

**Example** Show that the equation $x^{2008} + 2008x - 1$ has a root.
A root of the equation $f(x) = x^{2008} + 2008x - 1$ is a solution to the equation $f(x) = 0$,so we have to prove that f(x) = 0 has a solution.
First, we notice that f(x) is a continuous function, since f(x) is a polynomial, and polynomials are continuous. Further we can evaluate

$$f(0) = 0^{2008} + 2008 \cdot 0 - 1 = -1 < 0 = N$$
$$f(1) = 1^{2008} + 2008 \cdot 1 - 1 = 1 + 2008 - 1 = 2008 > 0 = N$$

Since f(x) is a continuous function on $[0, 1]$ with $f(0) < 0$ and $f(1) > 0$, we may apply the IVT and conclude that there exists a number c in the interval $(0, 1)$ with $f(c) = 0$ or $c^{2008} + 2008 \cdot c - 1 = 0$, which is exactly what we wanted to show.

Side NoteIndeed, the conclusion of the intermediate value theorem is somewhat mysterious in that it states the existence of a number c which has a nice property f(c) = N but does not actually tell you what that number is. This is about the only rule we have to show that a certain number exists without actually producing it, so if someone asks you to show a certain number exists but does not make you actually find it, you should start thinking intermediate value theorem.

Heres a trickier example that is a good example of this principle

**Example** Show that there exists a number x in the interval $[0, \frac{\pi}{2}]$ with $x = \cos(x)$.

This problem asks us to show there exists a number x which has a nice property (this time, nice property means $x = \cos(x)$) but does not ask us to say exactly what it is, so we are thinking that we should use the intermediate value theorem.

The first thing we need to do is translate this problem into a problem that we can use the intermediate value theorem to solve. In particular, given a continuous function g(x) and some hypotheses, the intermediate value theorem lets us conclude the existence of a number c with $g(c) = N$, where N is some real number. So lets translate our problem into the solution of an equation.