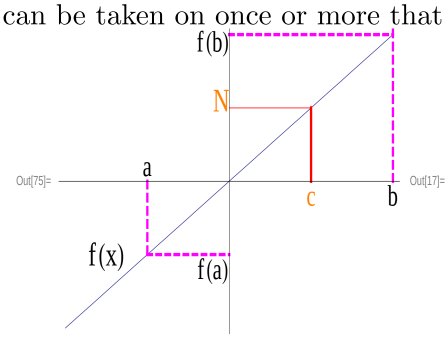
The intermediate Value Theorem(IVT)

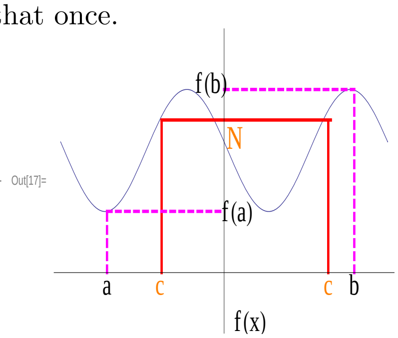
Theorem Suppose that f is continuous on the closed interval $[a, b]$ and either;
- $f(a) < N$ and $f(b) > N$ or
- $f(a) > N$ and $f(b) < N$

Then there is a number c in the interval (a, b) for which $f(c) = N$.

Remarks: 1)In geometric terms IVT says that if any horizontal line $y = N$ is given between $y = f(a)$ and $y = f(b)$ then graph of f cannot jump over the line. It must intersect $y = N$ somewhere

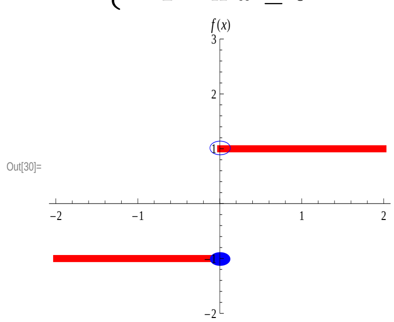
2) The IVT states that there is such a number c it neither says what this c is nor claims c to be unique. It is shown in the figures below that the value N can be taken on once or more that once.





3) It is important that the function f in the theorem be continuous, IVT is not true in general for discontinuous functions as can be seen in the example below.

$$f(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x \leq 0 \end{cases}$$



Note that there are no c-values on $[-1, 1]$ where $-1 < f(c) = 1/2 < 1$ or any other N where $-1 < N < 1$ for that matter.