

exponential functions, log functions (on their domain), and the absolute value function are all continuous.

We can also put together two continuous functions and get continuous functions: the sum, difference, product, scaling, quotient, and composition of two continuous functions are all continuous on the domain of the resultant function!

Example Find $\lim_{x \rightarrow e^7/\pi} \sin(x)$?

Since $\sin(x)$ is continuous by definition of continuity, $\lim_{x \rightarrow e^7/\pi} \sin(x) = \sin(e^7/\pi)$

Since continuity is defined as a limit, the limit laws have a direct correlation to continuous functions.

Theorem If f and g are continuous at $x=a$ and c is a constant, then the following are also continuous at $x=a$

1) $f \pm g$

2) cf

3) $(f \cdot g)$

4) If $g(a) \neq 0$ $\frac{f}{g}$

Example Where is $f(x) = \frac{\ln x + \tan^{-1}(x)}{x^2 - 1}$ continuous?

Divide and conquer: $\ln(x)$ is continuous on its domain $(0, \infty)$ and $\tan^{-1}(x)$ is continuous on all of \mathfrak{R} . Like wise $x^2 - 1$. By the property (1) above the numerator $\ln(x) + \tan^{-1}(x)$ continuous on $(0, \infty)$. By Property (4) $f(x)$ is continuous on $(0, \infty)$ where $x^2 - 1 \neq 0$. Hence $f(x)$ is continuous on $(0, 1) \cup (1, \infty)$

Theorem If f is continuous at b and $\lim_{x \rightarrow a} g(x) = b$, then $\lim_{x \rightarrow a} f(g(x)) = f(b)$. In other words;

$$\lim_{x \rightarrow a} f(g(x)) = f(\lim_{x \rightarrow a} g(x))$$