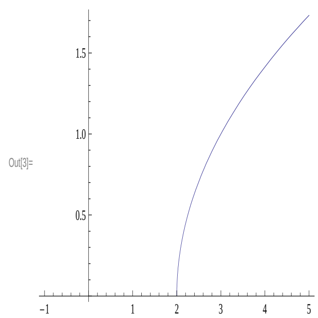
Example Recall we have showed using the Squeeze Theorem $\lim_{x\to 0} x^2 \sin(\frac{1}{x}) = 0$. Clearly $f(0)$ is not defined. So this function is a discontinuous function. But since the limit exists at $x = 0$, we can "fill in the hole" by defining a new function h(x) as follows $h(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$
h(x) now is continuous at $x = 0$ $\lim_{x\to 0} h(x) = h(0)$ so we have *removed* the discontinuity.

Definition A function is <u>continuous from the right</u> at $x = a$ if $\lim_{x\to a^+} f(x) = f(a)$

Example $f(x) = \sqrt{x-2}$. Applying the Pencil Test to the graph of f(x) we see that f is continuous at every point in $(2, \infty)$



Also we have previously showed that $\lim_{x\to 2^+} f(x) = 0$. We say that f is continuous on its domain $[2, \infty)$. Closed bracket at 2 implies that the function is continuous from right.

Definition A function f is <u>continuous on an interval</u> $[a, b]$ if it is continuous at every point in the interval. At the end points of the interval "a" and "b" by continuity we understand one sided-continuity. At "a" we require continuity from right and at "b" we require continuity from left.

Remark If a function is continuous on all of $\Re$ it is referred to simply as continuous.

Example Let p(x) be any polynomial. In Section 1.3 we have showed that $\lim_{x\to a} p(x) = p(a)$ at all a in $\Re$. So by definition above p(x) is continuous.

In fact, almost all the functions that we have talked about in our library are continuous: polynomials, rational functions (on their domain –so in particular where the denominator is not 0), trig functions (on their domain),