Definition A function f is continuous at $x = a$ if

1) $lim_{x \to a} f(x)$ exists

2) $f(a)$ is defined

3) $\lim_{x \to a} f(x) = f(a)$

Remark: If f(x) is not continuous at $x = a$, it is discontinuous at $x = a$.

There are a few important things to notice about continuous functions:

1) evaluating limits of continuous functions is easy, because its the same as evaluating the function;

2) a function can only be continuous on its domain, since the definition involves evaluating f at a (in particular if f(a) is not defined, then f(x) cant be continuous at a);

3) a continuous function has a limit at a (in particular, if $\lim_{x \to a} f(x)$ does not exist, f cant be continuous).

Types of discontinuity A function can fail to be continuous in a few different ways. The two big ways well see for a function to fail to be continuous at a point are jump discontinuities and removable discontinuities.
• A jump discontinuity is a point a so that $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ both exist, but are not equal.

• A removable discontinuity is a point a so that $lim_{x \to a} f(x)$ exists, but $lim_{x \to a} f(x) \neq f(a)$.
Jump discontinuities are irredeemable, in the sense that unless we give a major make-over to the graph of $f(x)$ we cannot get rid of them. However, Removable discontinuities needs only fixing at only one point, namely at "a" . We can remove this type of discontinuity and get a continuous function. Here is how such an operation is done on an example: