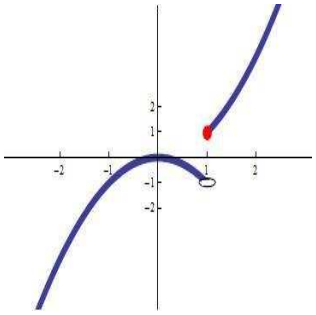
Case 3 Jump Discontinuity

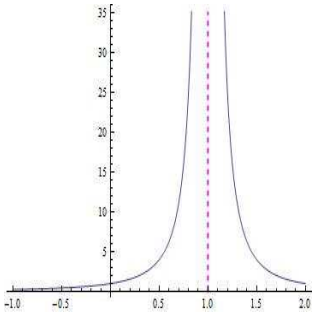
Consider the graph of $f(x) = \begin{cases} x^2 & \text{if } x \geq 1 \\ -x^2 & \text{if } x < 1 \end{cases}$



$f(1)$ exists but $\lim_{x \to 1} f(x)$ does not exist.

Case 4 Vertical Asymptotes

Consider $f(x) = \frac{1}{(x-1)^2}$



Here $f(x)$ grows unbounded as $x \to 1^+$ or $x \to 1^-$. Hence $f(1)$ does not exist and $\lim_{x \to 1} f(x)$ does not exist.

So based on our observations above the below definition should not come as a surprise.