

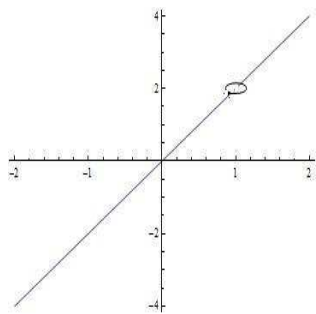
Sections 1.4 Continuity

Intuitive Definition or Pencil Test If you can trace the graph of a function without lifting your pencil, the function is continuous.

For motivation we will consider graphically 4 fundamental discontinuities:

Case 1: $f(a)$ is not defined – a hole in the graph case.

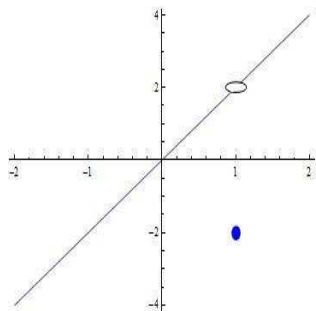
Consider the graph of the function $f(x) = \frac{x^2-1}{x-1}$



Note that $\lim_{x \rightarrow 1} f(x)$ exists but $f(1)$ is not defined.

Case 2 $f(a)$ is defined but still a hole in the graph.

Consider the graph of $f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x \neq 1 \\ -1 & \text{if } x = 1 \end{cases}$



Note that $\lim_{x \rightarrow 1} f(x)$ exists and $f(1)$ is defined but $\lim_{x \rightarrow 1} f(x) \neq f(1)$