**Theorem** For any number a, we have

| | |
|---|---|
| $\lim_{x\to a} \sin(x) = \sin(a)$ | $\lim_{x\to a} \sin^{-1}(x) = \sin^{-1}(a)$ for $-1 < a < 1$ |
| $\lim_{x\to a} \cos(x) = \cos(a)$ | $\lim_{x\to a} \cos^{-1}(x) = \cos^{-1}(a)$ for $-1 < a < 1$ |
| $\lim_{x\to a} e^x = e^a$ | $\lim_{x\to a} \tan^{-1}(x) = \tan^{-1}(a)$ for $-\infty < a < \infty$ |
| $\lim_{x\to a} \ln(x) = \ln(a)$, for $a > 0$ | |
| If p(x) is a polynomial, and $\lim_{x\to p(a)} f(x) = L$ then $\lim_{x\to a} f(p(x)) = L$ | |

We will re-visit the results of this theorem when we deal with "continuity" in the next section. Now let's learn another tool that will help us to evaluate some important limits such as $\lim_{x\to 0} \frac{\sin(x)}{x}$

**Theorem** If $f(x) \leq g(x)$ when x is near "a" (except possibly at a) and the limits of f and g exists as x approaches a, then

$$\lim_{x\to a} f(x) \leq \lim_{x\to a} g(x)$$

**Squeeze Theorem/Sandwich Theorem** If $f(x) \leq g(x) \leq h(x)$ when x is near "a" (except for possibly at a) and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$ then,

$$\lim_{x\to a} g(x) = L$$

**Example** Show that $\lim_{x\to 0} x^2 \sin(\frac{1}{x}) = 0$.

First note that you cannot use the limit law 3 for products of functions because $\lim_{x\to 0} \sin(1/x)$ does not exist. However, since

$$-1 \leq \sin(\frac{1}{x}) \leq 1 \text{ for all } x \neq 0$$

we have

$$-x^2 \leq x^2 \sin(\frac{1}{x}) \leq x^2 \text{ since } x^2 \geq 0 \text{ always.}$$