Example Find $\lim_{x\to -2}\sqrt{x^4+3x+6}$.

$$\lim_{x\to -2}\sqrt{x^4+3x+6} = \sqrt{\lim_{x\to -2}(x^4+3x+6)} = \sqrt{(-2)^2+3(-2)+6} = 4$$

Example Find $\lim_{x\to 2}\sqrt{x-2}$.

Your first reaction to this problem should be that limit does not exits. Because any open interval around x=2 contains values of x not in the domain of $\sqrt{x-2}$. But if we correct the question and discuss the limit from the right we have an answer:
$\lim_{x\to 2^+}\sqrt{x-2}=0$. If you consider $\lim_{x\to 2^-}\sqrt{x-2}$ does not exist. By a fact we have seen before you can also conclude that $\lim_{x\to 2}\sqrt{x-2}$ does not exist.

Some Cautionary Examples: Note that I have underlined the verb "exists" in the statement of the limit laws. You have to be cautious of this fact when you do the limit calculations. If either one of the limits do not exists, then you cannot automatically assume that you are able to use these rules. The Limit Laws guarantee results only if both results exits. So natural question might come to your mind such as:

<u>Question</u> Does $\lim_{x\to a}[f(x)+g(x)]$ exits even though $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ does not exists?
<u>Answer</u> It depends on $f(x)$ and $g(x)$. Check out the following:

Let $f(x) = \begin{cases} 0 & x<0 \\ 1 & x\geq 0 \end{cases}$
and $g(x) = \begin{cases} 1 & x<0 \\ 0 & x\geq 0 \end{cases}$

Both $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist because both of their one-sided limits are not equal. $\lim_{x\to 0^+} f(x) = 1 \neq \lim_{x\to 0^-} f(x) = 0$ and $\lim_{x\to 0^+} g(x) = 0 \neq \lim_{x\to 0^-} g(x) = 1$
But $f(x)+g(x)=1$ for all x. So $\lim_{x\to 0}[f(x)+g(x)]=1$ hence the limit of the sum exists.