Example Find $\lim\limits_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2}$.

We cannot simply substitute $x = 0$ since $f(0)$ is not defined. Hence we cannot apply the Law 4 for quotients (denominator is zero). However we can rationalize the numerator as follows:

$$\begin{aligned}
\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} &= \lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} \frac{\sqrt{x^2+9}+3}{\sqrt{x^2+9}+3} \\
&= \lim_{x \to 0} \frac{(x^2+9)-9}{x^2(\sqrt{x^2+9}+3)} = \lim_{x \to 0} \frac{x^2}{x^2(\sqrt{x^2+9}+3)} \\
&= \lim_{x \to 0} \frac{1}{(\sqrt{x^2+9}+3)} = \frac{1}{\sqrt{\lim\limits_{x \to 0}(x^2+9)}+3} \\
&= \frac{1}{3+3} = \frac{1}{6}
\end{aligned}$$

Example Find $\lim\limits_{x \to 2} \frac{x^2-4}{x-2}$

Note that again we cannot use the Law 4 for quotients because the denominator is zero at $x = 2$. So we need another algebra trick;

$$\begin{aligned}
\lim_{x \to 2} \frac{x^2-4}{x-2} &= \lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)} \\
&= \lim_{x \to 2}(x+2) = 4
\end{aligned}$$

In the above example note that $f(x) \neq x + 2$. The rational function is very much like $x + 2$ except it is not defined at $x = 2$. So we are using the power of the limit.

Theorem Let $\lim\limits_{x \to a} f(x) = L$ and n is any positive integer, then

$$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$$

If n is even, we need $\lim\limits_{x \to a} f(x) > 0$.