Sections 1.3 Computation of Limits

We will shortly introduce the "limit laws." Limit laws allows us to evaluate the limit of more complicated functions using the limit of simpler ones.
Theorem Suppose that "c" is a constant and the limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exists. Then;

1)

$$\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$$

2)

$$\lim_{x\to a}[cf(x)] = c \lim_{x\to a} f(x)$$

3)

$$\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$

4)

$$\text{If } \lim_{x\to a} g(x) \neq 0, \text{ then } \lim_{x\to a}[\frac{f(x)}{g(x)}] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$$

5)

$$\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n \text{ n is a positive integer}$$

Indeed the same rules hold for directional limits, so that (for instance) if $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^-} g(x)$ exists, then

$$\lim_{x\to a^-}[f(x) + g(x)] = \lim_{x\to a^-} f(x) + \lim_{x\to a^-} g(x)$$

Corollary If f(x) is a polynomial, then $\lim_{x\to a} f(x) = f(a)$.

Proof: If f(x) is a polynomial, we know that $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$. Then we have