Intuitive Definition For a function f(x), *the limit of f as x approaches a from the left*, written $\lim_{x\to a^-} f(x)$ is the quantity that outputs are approaching as inputs approach a from the left. The *limit of f as x approaches a from the right*, written $\lim_{x\to a^+} f(x)$, is the quantity that outputs are approaching as inputs approach a from the right.

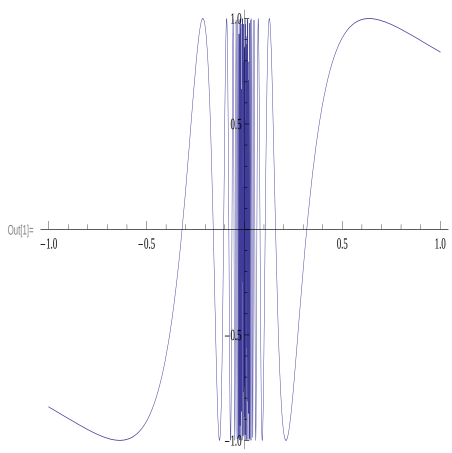
Example Consider the function f(x) from the previous example. What is $\lim_{x\to 0^-} f(x)$ and $\lim_{x\to 0^+} f(x)$?
We said above that f(x) approaches 1 and -1 as inputs approach 0 from the right and left (respectively). Hence we have,

$$\lim_{x\to 0^-} f(x) = -1 \text{ and } \lim_{x\to 0^+} f(x) = 1$$

Fact: $\lim_{x\to a} f(x)$ exists if and only if $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ exist and are equal.

This fact comes in handy for evaluating limits of functions that look pretty tricky.We'll see an example of this soon.

Example The function $y = \sin(\frac{1}{x})$ is another example of a "limit does not exist case".



Consider the limit $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$. This function does not even have a directional limit at 0!. You can see from the graph that as $x \to 0^+$, the function is not approaching a single value. Indeed, for any value c in the interval $[-1, 1]$ that you like, there is a sequence of points approaching 0 whose outputs are approaching to c. In this sense, then the function fails to have directional limit because it is approaching "too many" values as $x \to 0^+$