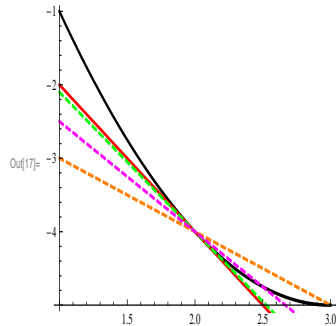


graph below.



So by this graphical observation we can conclude that the slope of the secant lines will start to approach the slope of the tangent line. Hence, if we can see what happens to the quantity

$$m = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

as  $Q$  approaches  $P$ , or equivalently as  $x_1$  approaches  $x_0$ , we will have the slope of the tangent line as desired. Soon we will be writing "what happens to the quantity  $\frac{f(x_1) - f(x_0)}{x_1 - x_0}$  as  $x_1$  approaches  $x_0$  as:

$$\lim_{x_1 \rightarrow x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

We will call this quantity the derivative of  $f(x)$  at  $x_0$ , which we will abbreviate as  $f'(x_0)$

To see a rigorous example of this let's go back to our parabola  $x^2 - 6x + 4$  and check the table values below (Note that this time I have used  $x_1$  values less than 2)

$x_1$	$f(x_1)$	$m_{sec}$
1.5	-2.75	$\frac{-2.75+4}{1.5-2} = -2.5$
1.9	-3.79	$\frac{-3.79+4}{1.9-2} = -2.1$
1.99	-3.9799	$\frac{-3.9799+4}{1.99-2} = -2.01$

By comparing the two tables we observe even though the function  $m_{sec}(x) = \frac{f(x)+4}{x-2}$  is not defined at  $x = 2$  it appears that as we approach  $x=2$  from right or left (on the  $x$ -axis)  $m_{sec}(x)$  is going to the value -2. So the slope of the tangent line at  $(2, -4)$  is -2.