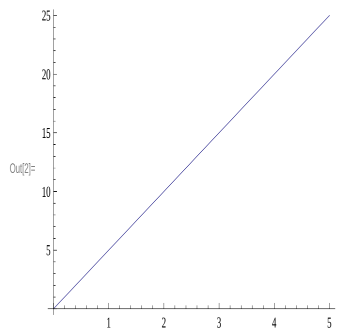
Sections 1.1/1.2 Intro to Calculus and Limit

The distance one travels at a constant speed had long been modeled as a linear function. If we are traveling at a constant velocity m, then the total distance traveled from time 0 to time x is $f(x) = mx$



If you pick any two arbitrary points from the line and calculate the difference quotient that gives you the slope; it will give you the same answer regardless of the choice of points you go with. And the slope of the line is equal to the constant speed.

Question: Why wouldn't the same geometry apply to a variable speed problem?

The answer hinges on the definition of the *slope of a curve* at a point first. For instance, suppose we wanted to find the slope of the curve $y = f(x)$ at the point $P = (x_0, f(x_0))$. –This corresponds to finding the speed of the object at time $x = x_0$ with the displacement function $f(x)$ btw– how is the slope defined? Like everything else in math start with what we know and let's see how can we extend this into a new definition. To see what is going on let's pick up a function, say $f(x) = x^2 - 6x + 4$ and a point of interest, say $P = (x_0, y_0) = (2, -4)$. The big idea is to consider **secant lines**. Specifically, pick a point $Q = (x_1, f(x_1))$ on the graph of f(x) and find the slope of the line through P and Q (you can do this since 2 points determine a line). This is the so called secant line through P and Q.