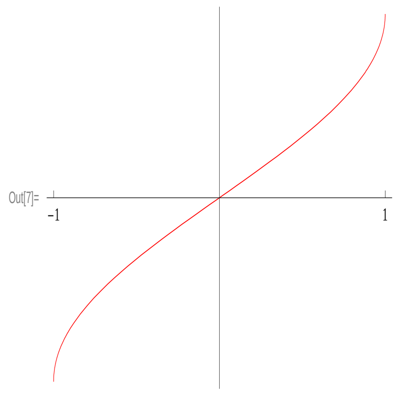

Note that $\sin x : [-\pi/2, \pi/2] \to [-1, 1]$ and $\arcsin x : [-1, 1] \to [-\pi/2, \pi/2]$. Since the definition of an inverse function says that

$$f^{-1}(x) = y \Leftrightarrow f(y) = x$$

so we have

$$\sin^{-1} x = y \Leftrightarrow \sin y = x \text{ and } \frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$$

or $\sin(\sin^{-1} x) = x$ for $x \in [-1, 1]$ and $\sin^{-1}(\sin x) = x$ for $x \in [-\pi/2, \pi/2]$

Warning Example $\sin^{-1}(\sin \pi) = \sin^{-1}(0) = 0$. π is not in the domain of arcsin so not ending up with π is not a surprise.

Example Evaluate $\sin^{-1}(\frac{\sqrt{3}}{2})$

By the definition of $\sin^{-1}$ we want an angle $\theta \in [-\pi/2, \pi/2]$ where $\sin \theta = \frac{\sqrt{3}}{2}$ so $\theta = \frac{\pi}{3}$.

Example Evaluate $\sin^{-1}(\frac{-1}{2})$
Find θ s.t $\sin \theta = \frac{-1}{2}$ so $\theta = \frac{-\pi}{6}$

Example Find the domain of f(x) for $f(x) = \sin^{-1}(\frac{1}{x})$.

Domain of arcsin is $[-1, 1]$ so we are interested in those real numbers x such that $-1 \leq \frac{1}{x} \leq 1$. And this inequality is satisfied if $x \in (-\infty, -1] \cup [1, \infty)$. So Domain of f=$(-\infty, -1] \cup [1, \infty)$

Example Find the inverse of $y = \cos x$. Cosine is, like sine, not 1-1 so we cannot find the inverse of it unless we restrict our domain. We will restrict our domain to be $[0, \pi]$. As you can check from the graph of $\cos x$ is