

University of Illinois at Urbana-Champaign
Dept. of Electrical and Computer Engineering

ECE 220: Computer Systems & Programming

Recursion

Writing the Fibonacci Sequence

Anyone remember the Fibonacci sequence?

1, 1, 2, 3, 5, 8, 13, ...

Can anyone write the whole sequence?

(The rest of us can be done for today!)

How about this way: $F(0) = 1$
 $F(1) = 1$
 $F(N) = F(N - 2) + F(N - 1)$

This answer is a **recursive definition**, a **function defined in terms of itself**.

Fibonacci Sequence is Well-Defined

Fibonacci: $F(0) = 1$
 $F(1) = 1$
 $F(N) = F(N - 2) + F(N - 1)$

Given this definition, we say that $F(N)$

- is **well-defined** because
- it **eventually stops recursing** for all $N \geq 0$,
- or, **equivalently**, $F(N)$ satisfying the equations **is unique** for all $N \geq 0$.

Some Sequences are Not Well-Defined

This sequence is not well-defined:

$G(0) = 1$
 $G(N) = [G(N - 1) + G(N + 1)] / 2$

What can $G(N)$ be?

1, 1, 1, 1, 1, 1, 1, ...

1, 2, 3, 4, 5, 6, 7, ...

The possibilities are infinite.

$G(N)$ is not well-defined.