

Lack of Causality

But if $u = K\dot{e}$, then $U = KsE \implies \frac{U}{E} = Ks = \frac{q(s)}{p(s)}$

$\deg(q) > \deg(p)$ — *improper system* (lack of causality)

So, $E \mapsto K_D s E$ is not implementable directly, but we can implement an approximation, e.g.

$$\frac{K_D a s}{a + s} \longrightarrow K_D s \quad \text{as } a \rightarrow \infty$$

(this can be done using op-amps).

Alternatively, we can approximate derivative action using finite differences:

$$\dot{e}(t) \approx \frac{e(t + \delta) - e(t)}{\delta},$$

but then we must tolerate delays — must wait until time $t + \delta$ to issue a control signal meant for time t .