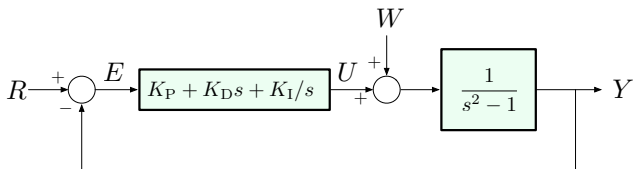# Proportional-Integral-Derivative (PID) Control

$$Y = \frac{1}{s^2 - 1}(U + W), \qquad U = \left(K_P + K_D s + \frac{K_I}{s}\right)(R - Y)$$

$$\text{so } Y = \frac{K_P + K_D s + \frac{K_I}{s}}{s^2 - 1}(R - Y) + \frac{1}{s^2 - 1}W$$

Simplify:

$$(s^2 - 1)Y = \left(K_P + K_D s + \frac{K_I}{s}\right)(R - Y) + W$$

$$\left(s^2 - 1 + K_P + K_D s + \frac{K_I}{s}\right)Y = \left(K_P + K_D s + \frac{K_I}{s}\right)R + W$$

$$(s^3 - s + K_P s + K_D s^2 + K_I)Y = (K_P s + K_D s^2 + K_I)R + Ws$$