Marginal Case: Poles on the Imaginary Axis

Let's consider the case of a pole at the origin: $H(s) = \dfrac{1}{s}$

Is this a stable system?

- impulse response: $Y(s) = \dfrac{1}{s} \Longrightarrow y(t) = 1(t)$ (OK)
- step respone: $Y(s) = \dfrac{1}{s^2} \Longrightarrow y(t) = t,\ t \geq 0$ — *unit ramp*!!

What about purely imaginary poles? $H(s) = \dfrac{\omega^2}{s^2 + \omega^2}$

- impulse response: $Y(s) = \dfrac{\omega^2}{s^2 + \omega^2} \Longrightarrow y(t) = \omega \sin(\omega t)$
- step respone: $Y(s) = \dfrac{\omega^2}{s(s^2 + \omega^2)} \Longrightarrow y(t) = 1 - \cos(\omega t)$

Systems with poles on the imaginary axis are *not stable*.