

Effect of a LHP Zero

$$H_1(s) = \frac{1}{s^2 + 2\zeta s + 1} \xrightarrow{\text{add zero at } s = -a} H_2(s) = H_1(s) + \frac{1}{a} \cdot sH_1(s)$$

Step response:

$$\begin{aligned} Y_1(s) &= \frac{H_1(s)}{s} \\ Y_2(s) &= \frac{H_2(s)}{s} \\ &= \frac{H_1(s)}{s} + \frac{1}{a} \frac{sH_1(s)}{s} \\ &= Y_1(s) + \frac{1}{a} sY_1(s) \end{aligned}$$

$$y_2(t) = \mathcal{L}^{-1}\{Y_2(s)\} = \mathcal{L}^{-1}\left\{Y_1(s) + \frac{1}{a} \cdot sY_1(s)\right\} = y_1(t) + \frac{1}{a} \dot{y}_1(t)$$

(assuming zero initial conditions)