

The Routh–Hurwitz Criterion

Consider degree- n polynomial

$$p(s) = s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n$$

and form the Routh array:

$$\begin{array}{rcccccc} s^n & : & 1 & a_2 & a_4 & a_6 & \dots \\ s^{n-1} & : & a_1 & a_3 & a_5 & a_7 & \dots \\ s^{n-2} & : & b_1 & b_2 & b_3 & \dots & \\ s^{n-3} & : & c_1 & c_2 & \dots & & \\ & & \vdots & & & & \\ s^1 & : & * & * & & & \\ s^0 & : & * & & & & \end{array}$$

The Routh–Hurwitz criterion: Assume that the necessary condition for stability holds, i.e., $a_1, \dots, a_n > 0$. Then:

- ▶ p is stable if and only if all entries in the first column are positive;
- ▶ otherwise, $\#(\text{RHP poles}) = \#(\text{sign changes in 1st column})$