Effect of Zeros on the Transient Response

Reminder: for $H(s) = \dfrac{q(s)}{p(s)}$, zeros are the roots of $q(s) = 0$

Example: start with $H_1(s) = \dfrac{1}{s^2 + 2\zeta s + 1}$ $\qquad (\omega_n = 1)$

Let's add a zero at $s = -a$, $a > 0$ $\qquad$ – LHP zero

To keep DC gain $= 1$, let's take the numerator to be $\dfrac{s}{a} + 1$:

$$H_2(s) = \frac{\frac{s}{a} + 1}{s^2 + 2\zeta s + 1}$$

$$= \underbrace{\frac{1}{s^2 + 2\zeta s + 1}}_{\text{this is } H_1(s)} + \frac{1}{a} \cdot \underbrace{\frac{s}{s^2 + 2\zeta s + 1}}_{\text{call this } H_d(s)}$$

$$= H_1(s) + \frac{1}{a} H_d(s), \qquad H_d(s) = sH_1(s)$$