

TD Specs in Frequency Domain

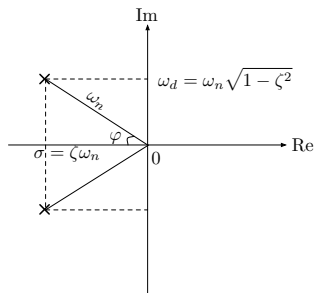
We want to *visualize* time-domain specs in terms of *admissible pole locations* for the 2nd-order system

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\sigma^2 + \omega_d^2}{(s + \sigma)^2 + \omega_d^2}$$

$$\text{where } \sigma = \zeta\omega_n$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

$$\text{Step response: } y(t) = 1 - e^{-\sigma t} \left(\cos(\omega_d t) + \frac{\sigma}{\omega_d} \sin(\omega_d t) \right)$$



$$\begin{aligned}\omega_n^2 &= \sigma^2 + \omega_d^2 \\ \zeta &= \cos \varphi\end{aligned}$$