

Prototype 2nd-Order System

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

By the quadratic formula, the poles are:

$$\begin{aligned} s &= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} \\ &= -\omega_n \left(\zeta \pm \sqrt{\zeta^2 - 1} \right) \end{aligned}$$

The nature of the poles changes depending on ζ :

- ▶ $\zeta > 1$ both poles are real and negative
- ▶ $\zeta = 1$ one negative pole
- ▶ $\zeta < 1$ two complex poles with negative real parts

$$s = -\sigma \pm j\omega_d$$

where $\sigma = \zeta\omega_n$, $\omega_d = \omega_n\sqrt{1 - \zeta^2}$