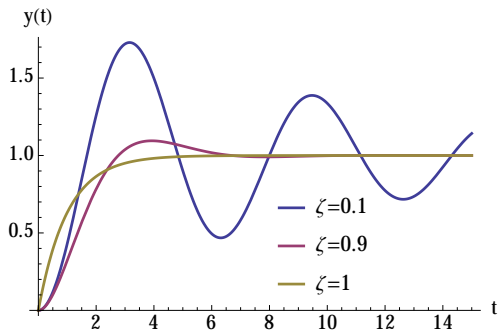


## 2nd-Order Step Response

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s + \sigma)^2 + \omega_d^2}$$

$$u(t) = 1(t) \quad \rightarrow \quad y(t) = 1 - e^{-\sigma t} \left( \cos(\omega_d t) + \frac{\sigma}{\omega_d} \sin(\omega_d t) \right)$$

where  $\sigma = \zeta\omega_n$  and  $\omega_d = \omega_n\sqrt{1 - \zeta^2}$  (damped frequency)



The parameter  $\zeta$  is called the *damping ratio*

- ▶  $\zeta > 1$ : system is overdamped
- ▶  $\zeta < 1$ : system is underdamped
- ▶  $\zeta = 0$ : no damping ( $\omega_d = \omega_n$ )