

2nd-Order Response

Let's compute the system's impulse and step response:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s + \sigma)^2 + \omega_d^2}$$

► Impulse response:

$$\begin{aligned}h(t) &= \mathcal{L}^{-1}\{H(s)\} = \mathcal{L}^{-1}\left\{\frac{(\omega_n^2/\omega_d)\omega_d}{(s + \sigma)^2 + \omega_d^2}\right\} \\ &= \frac{\omega_n^2}{\omega_d} e^{-\sigma t} \sin(\omega_d t) \quad (\text{table, \# 20})\end{aligned}$$

► Step response:

$$\begin{aligned}\mathcal{L}^{-1}\left\{\frac{H(s)}{s}\right\} &= \mathcal{L}^{-1}\left\{\frac{\sigma^2 + \omega_d^2}{s[(s + \sigma)^2 + \omega_d^2]}\right\} \\ &= 1 - e^{-\sigma t} \left(\cos(\omega_d t) + \frac{\sigma}{\omega_d} \sin(\omega_d t)\right) \quad (\text{table, \#21})\end{aligned}$$