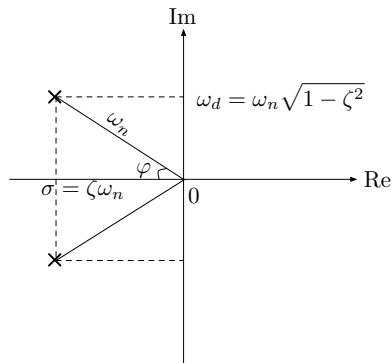# Prototype 2nd-Order System

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \qquad \zeta < 1$$

The poles are

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -\sigma \pm j\omega_d$$

Note that

$$\sigma^2 + \omega_d^2 = \zeta^2\omega_n^2 + \omega_n^2 - \zeta^2\omega_n^2$$
$$= \omega_n^2$$
$$\cos\varphi = \frac{\zeta\omega_n}{\omega_n} = \zeta$$