Prototype 2nd-Order System

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

By the quadratic formula, the poles are:

$$s = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$
$$= -\omega_n\left(\zeta \pm \sqrt{\zeta^2 - 1}\right)$$

The nature of the poles changes depending on ζ:

- $\zeta > 1$ both poles are real and negative
- $\zeta = 1$ one negative pole
- $\zeta < 1$ two complex poles with negative real parts

$$s = -\sigma \pm j\omega_d$$

$$\text{where} \quad \sigma = \zeta\omega_n, \ \omega_d = \omega_n\sqrt{1 - \zeta^2}$$