Method of Partial Fractions

Problem: compute $\mathscr{L}^{-1}\{Y(s)\}$, where

$$Y(s) = \frac{s}{(s+1)(s^2+1)}$$

We found that

$$Y(s) = -\frac{1}{2(s+1)} + \frac{s}{2(s^2+1)} + \frac{1}{2(s^2+1)}$$

Now we can use linearity and tables:

$$\begin{aligned}
y(t) &= \mathscr{L}^{-1}\left\{-\frac{1}{2(s+1)} + \frac{s}{2(s^2+1)} + \frac{1}{2(s^2+1)}\right\} \\
&= -\frac{1}{2}\mathscr{L}^{-1}\left\{\frac{1}{s+1}\right\} + \frac{1}{2}\mathscr{L}^{-1}\left\{\frac{s}{s^2+1}\right\} + \frac{1}{2}\mathscr{L}^{-1}\left\{\frac{1}{s^2+1}\right\} \\
&= -\frac{1}{2}e^{-t} + \frac{1}{2}\cos t + \frac{1}{2}\sin t \quad \text{(from tables)} \\
&= -\frac{1}{2}e^{-t} + \frac{1}{\sqrt{2}}\cos(t - \pi/4) \quad (\cos(a-b) = \cos a \cos b + \sin a \sin b)
\end{aligned}$$