

Method of Partial Fractions

Problem: compute $\mathcal{L}^{-1}\{Y(s)\}$, where

$$Y(s) = \frac{s}{(s+1)(s^2+1)}$$

We seek a, b, c , such that

$$Y(s) = \frac{a}{s+1} + \frac{bs+c}{s^2+1} \quad (\text{need } bs+c \text{ so that } \deg(\text{num}) = \deg(\text{den}) - 1)$$

► Find b : multiply by s^2+1 to isolate $bs+c$

$$(s^2+1)Y(s) = \frac{s}{s+1} = \frac{a(s^2+1)}{s+1} + bs+c$$

— now let $s=j$ to “kill” the first term on the RHS:

$$bj+c = (s^2+1)Y(s)\Big|_{s=j} = \frac{j}{1+j}$$

Match $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ parts:

$$c+bj = \frac{j}{1+j} = \frac{j(1-j)}{(1+j)(1-j)} = \frac{1}{2} + \frac{j}{2} \implies b=c = \frac{1}{2}$$