Laplace Transforms Revisited

Example: $f(t) = \cos t$

$$\mathscr{L}\{\cos t\} = \mathscr{L}\left\{\frac{1}{2}e^{jt} + \frac{1}{2}e^{-jt}\right\} \qquad \text{(Euler's formula)}$$

$$= \frac{1}{2}\mathscr{L}\{e^{jt}\} + \frac{1}{2}\mathscr{L}\{e^{-jt}\} \qquad \text{(linearity)}$$

$$\mathscr{L}\{e^{jt}\} = \int_0^\infty e^{jt}e^{-st}\mathrm{d}t = \int_0^\infty e^{(j-s)t}\mathrm{d}t = \frac{1}{j-s}e^{(j-s)t}\Big|_0^\infty$$

$$= -\frac{1}{j-s} \qquad \text{(pole at } s = j\text{)}$$

$$\mathscr{L}\{e^{-jt}\} = \int_0^\infty e^{-jt}e^{-st}\mathrm{d}t = \int_0^\infty e^{-(j+s)t}\mathrm{d}t = -\frac{1}{j+s}e^{-(j+s)t}\Big|_0^\infty$$

$$= \frac{1}{j+s} \qquad \text{(pole at } s = -j\text{)}$$

— in both cases, require $\mathrm{Re}(s) > 0$, i.e., s must lie in the right half-plane (RHP)