The Final Value Theorem

We can now deduce the Final Value Theorem (FVT):

If all poles of $sY(s)$ are *strictly stable* or lie in the *open left half-plane* (OLHP), i.e., have $\mathrm{Re}(s) < 0$, then

$$y(\infty) = \lim_{s \to 0} sY(s).$$

In our examples, multiply $Y(s)$ by s, check poles:

- $Y(s) = \dfrac{1}{s+a} \qquad sY(s) = \dfrac{s}{s+a}$
  if $a > 0$, then $y(\infty) = 0$; if $a < 0$, FVT does not give correct answer
- $Y(s) = \dfrac{1}{s^2+\omega^2} \qquad sY(s) = \dfrac{s}{s^2+\omega^2}$
  poles are purely imaginary (not in OLHP), FVT does not give correct answer
- $Y(s) = \dfrac{c}{s} \qquad sY(s) = c$
  poles at infinity, so $y(\infty) = c$ – FVT gives correct answer